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RESPONSE TO VIBRATIONAL DISTURBANCE

OF THE MAGNET FOUNDATION

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Response to Vibrational Disturbance of the Magnet Foundation

Assumptions:

1. Soil under the concrete slab of uniform density and elasticity (constant ρ and constant modulus of elasticity E).
2. Constant frictional damping.
3. No coupling between vertical and horizontal motion.

Vertical Motion

Hooke's law: $$\frac{z - z_o}{z_o} = \frac{\text{Tension}}{E} = \frac{\text{Force}}{AE}$$

A = area of the slab

Force: $$F = -Mg - M\frac{d^2z}{dt^2} - F_f + F_d$$

g = gravity acceleration

F_f = frictional force $\approx C\frac{dz}{dt}$

F_d = driving force (external and/or internal)

= $f \cos \omega t$

M = $M_1 + A(L - z_o)\rho$

M_1 = mass of concrete slab + magnets

L = distance of slab to bedrock (assumed constant)

ρ = density of soil

Substitution of the force F in Hooke's law gives

$$\frac{z - z_o}{z_o} = -\frac{Mg - M\frac{d^2z}{dt^2} - C\frac{dz}{dt} + f \cos wt}{AE}$$

Setting $z - z_o + \frac{Mgz_o}{AE} = y$, $\frac{dz}{dt} = \frac{dy}{dt}$ and

$\frac{d^2z}{dt^2} = \frac{d^2y}{dt^2}$ we obtain

for $z_o = L$ $(M = M_1)$

$$\frac{d^2y}{dt^2} + \lambda \frac{dy}{dt} + \Omega^2 y = \varepsilon \cos \omega t.$$

where $\lambda = \frac{C}{M_1}$, $\Omega^2 = \frac{AE}{M_1 L}$ and $\varepsilon = \frac{f}{M_1}$.

The solution of this equation can be written in the form

$$y = \frac{\varepsilon}{\sqrt{(\Omega^2 - \omega^2)^2 + (\lambda\omega)^2}} \cos (\omega t - \phi), \quad \tan \phi = \frac{\lambda\omega}{\Omega^2 - \omega^2}$$

We see that at resonance $(\omega = \Omega)$, the amplitude is limited only by the frictional damping (λ). Knowing E, L, f and λ will give a rough estimate of the effects of vibrational disturbances.

In the foregoing treatment, the soil underneath the foundation is approximated by a pile. A more realistic result may be obtained by considering the soil as an "elastic" half-space. The weight of the concrete slab and the magnets will compress the soil and introduce vertical and horizontal strain. The static displacement is given by

$$\Delta z = \frac{M_1 g(1 - \mu)}{4G\sqrt{A/\pi}}$$

Here, μ is the Poisson's ratio $\frac{\text{horizontal strain}}{\text{vertical strain}}$ and $G = \frac{E}{2(1+\mu)}$ in the shear modulus.

Comparing Δz with the corresponding displacement for the pile approximation $\frac{M_1 g\ell}{AE}$ we obtain the effective spring constant $\frac{4G}{1-\mu}\sqrt{\frac{A}{\pi}}$ and the resonant frequency $\Omega^2 = \frac{4G}{M_1(1-\mu)}\sqrt{\frac{A}{\pi}}$.

To estimate the damping, we assume that the damping constant C is proportional to the area A, the density ρ and the wave velocity $\sqrt{\frac{G}{\rho}}$. To make the ratio:

$\frac{\text{spring constant}}{\text{damping constant}}$ independent of the Poisson's ratio, we set

$$C = \frac{A\sqrt{G\rho}}{1-\mu} \quad \text{or} \quad \lambda = \frac{A\sqrt{G\rho}}{M_1(1-\mu)} .$$

Assuming: $\rho = 2 \times 10^3$ kg/m^3

$G = 400$ kg/cm$^2 \approx 4 \times 10^7$ N/m^2

$A = 5$ m^2

$M_1 = 1.2 \times 10^4$ kg

$\mu = 0.3$

we obtain $\Omega = 155$ sec^{-1} ($F \approx 25$ Hz)

$\lambda = 168$ sec^{-1}

Substitutions of Ω and λ in $A_y = \frac{\varepsilon}{\sqrt{(\Omega^2-\omega^2)^2 + (\lambda\omega)^2}}$ we find at resonance ($\Omega = \omega$)

$A_y = 4$ μm for $\varepsilon = 0.01$ g ≈ 0.1 m/sec^2.